AAE 637 Lab 9: Conditional logit post-estimation*

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Question 1(c)

For each fishing *mode*, you want:

$$P_{ij}\Delta = P_{ij}\left(cost_{mode} = cost_{mode} + 100\right) - P_{ij}\left(cost_{mode} = cost_{mode}\right), \forall i, j$$

$$\mathbf{P\Delta} = \begin{bmatrix} P_{11}\Delta & \cdots & P_{14}\Delta \\ \vdots & \ddots & \vdots \\ P_{N1}\Delta & \cdots & P_{N4}\Delta \end{bmatrix}$$

$$P_{ij} = \frac{\exp\{\mathbf{X}_{ij}\boldsymbol{\beta}\}}{\sum_{r=1}^{4} \exp\{\mathbf{X}_{ir}\boldsymbol{\beta}\}}$$

$$\mathbf{A} = \begin{bmatrix} \exp\{\mathbf{X}_{11}\boldsymbol{\beta}\} & \cdots & \exp\{\mathbf{X}_{14}\boldsymbol{\beta}\} \\ \vdots & \ddots & \vdots \\ \exp\{\mathbf{X}_{N1}\boldsymbol{\beta}\} & \cdots & \exp\{\mathbf{X}_{N4}\boldsymbol{\beta}\} \end{bmatrix}$$

A can be constructed by:

```
A = zeros(N,4)
for j=1:4
  rhsvar_j = rhsvar(mode_id==j,:);
  A(:,j) = exp( rhsvar_j * b0 )
end
```

Performing `sum(A,2)` will yield:

*prepared by Travis McArthur, UW-Madison (http://www.aae.wisc.edu/tdmcarthur/teaching.asp)

$$\mathbf{P_{denom}} = \begin{bmatrix} \sum_{r=1}^{4} \exp\{\mathbf{X}_{1r}\boldsymbol{\beta}\} \\ \vdots \\ \sum_{r=1}^{4} \exp\{\mathbf{X}_{Nr}\boldsymbol{\beta}\} \end{bmatrix}$$

`P_denom_rep = repmat(P_denom, 1, 4)`

$\mathbf{A} \oslash \mathbf{P_{denomrep}} = \mathbf{P}$, where $\oslash$ is element-by-element division.

To create $\mathbf{P\Delta}$, do the same with $cost_{mode} = cost_{mode} + 100$, perhaps with a different globally-defined `rhsvar` matrix.

Then `mean(P_delta)` will obtain the column means, which is the desired quantity.

For standard errors, since your function above return a vector of 4 elements, you will need to use:

`Grad(params, 'cost_mfx', 4)`

Question 1(d)

The Cauchy–Schwarz inequality can lead to a shortcut here.

Let X and Y be random variables. Then:

$|E(X \cdot Y)| \leq \sqrt{E(X^2)} \cdot \sqrt{E(Y^2)}$

This leads to:

$-\sigma_X \sigma_Y \leq \sigma_{XY} \leq \sigma_X \sigma_Y$

where σ_X is the standard deviation of X (defined similarly for Y) and σ_{XY} is the covariance of X and Y

The standard deviation of the difference between two random variables X and Y is:

$\sigma_{X-Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 - 2 \cdot \sigma_{XY}}$

The null hypothesis of $X = Y$ is least likely to be rejected when σ_{X-Y} is largest.

Hence, if σ_{XY} is unknown, the "worst case scenario" is where $\sigma_{XY} = -\sigma_X \sigma_Y$.

If the null hypothesis can be rejected in the worst case scenario, it can be rejected regardless of the value of the unknown σ_{XY}.

The "worst case scenario" t-stat is:

$$t = \frac{X - Y}{\sqrt{\sigma_X^2 + \sigma_Y^2 - 2 \cdot (-\sigma_X \sigma_Y)}}$$

Question 1(e)

From Greene 7th ed. page 806, the elasticity of the probability of choosing the jth alternative with respect to the kth attribute of choice m is:

$$\frac{\partial \ln P_{ij}}{\partial \ln x_{mk}} = x_{imk} \left[\mathbb{1}\left\{j=m\right\} - P_{im}\right] \beta_k$$

i is the observation index.

In our case, catch rate is the 2nd attribute and private boat is the 3rd choice, so $k = 2$ and $m = 3$

You want:

$$\begin{bmatrix} \frac{\partial \ln P_{11}}{\partial \ln x_{32}} & \cdots & \frac{\partial \ln P_{14}}{\partial \ln x_{32}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \ln P_{N1}}{\partial \ln x_{32}} & \cdots & \frac{\partial \ln P_{N4}}{\partial \ln x_{32}} \end{bmatrix}$$

How can we implement this?

Construct P_{im} in the same way that P_{ij} was constricted in part (c).

So $\mathbf{P}$ is $N \times 4$.

Then contruct x_{imk} as a matrix:

```
pboat_catch_rate = rhsvar(mode_id==3, 2)
```

```
pboat_catch_rate_mat = repmat(pboat_catch_rate, 1, 4)
```

$\mathbb{1}\left\{j=m\right\}$ can be constructed by creating an $N \times 4$ matrix of zeros and then replacing the 3rd column by ones.

β_k is just a scalar, so nothing special is necessary.