AAE 637 Lab 8: Homework Questions*

4/8/2015

Catching your mistakes in homeworks:

- Ballparking. Note that for hypothesis testing, LM $\leq$ LR $\leq$ Wald.
- Coding globally is coding dangerously - set up a safety harness.

Question 1(c)

(Observation i subscripts suppressed)

$\Pr(y=1|\mathbf{X}) = \Phi(\mathbf{X}\boldsymbol{\beta}) = \Phi\left(\sum_{j=1}^{J} X_j\beta_j\right)$

When one X_j is $\log(hhninc)$, the above expression is:

$\Phi\left(\sum_{j=1}^{J-1} X_j\beta_j + \log(hhninc)\,\beta_{\log inc}\right)$

The marginal effect of the kth variable on the probability that $y=1$ is $\frac{\partial \Pr(y=1|\mathbf{X})}{\partial X_k}$

The marginal effect of $hhninc$ is:

$\frac{\beta_{\log inc}}{hhninc} \cdot \phi\left(\sum_{j=1}^{J-1} X_j\beta_j + \log(hhninc)\,\beta_{\log inc}\right)$

Question 1(d)

Four equivalent expressions for the elasticity of $f(\cdot)$ with respect to x are:

1. $\frac{\partial \log(f(\cdot))}{\partial \log(x)}$
2. $\frac{\partial f(\cdot)}{\partial x} \cdot \frac{x}{f(\cdot)}$
3. $\frac{\partial \log(f(\cdot))}{\partial x} \cdot x$

*prepared by Travis McArthur, UW-Madison (http://www.aae.wisc.edu/tdmcarthur/teaching.asp)

4. $\dfrac{\partial f(\cdot)}{\partial \log(x)} \cdot \dfrac{1}{f(\cdot)}$

Since the both the expression for $\Pr(y=1|\mathbf{X})$ and *age* are in levels rather than logs, using (2) here is easiest. Hence the elasticity is:

$$\frac{\partial \Pr(y=1|\mathbf{X})}{\partial X_k} \cdot \frac{X_j}{\Pr(y=1|\mathbf{X})}$$

$$= \frac{\partial \Phi\left(\sum_{j=1}^{J} X_j \beta_j\right)}{\partial X_k} \cdot \frac{X_k}{\Phi\left(\sum_{j=1}^{J} X_j \beta_j\right)}$$

$$= \beta_k \cdot \phi\left(\sum_{j=1}^{J} X_j \beta_j\right) \cdot \frac{X_k}{\Phi\left(\sum_{j=1}^{J} X_j \beta_j\right)}$$

Question 1(f)

Let $\mathbf{X}$ be the $N \times J$ RHS matrix, where N is the number of observations and J is the number of RHS variables.

Define $\mathbf{m} = \beta_k \cdot \phi(\mathbf{X}\boldsymbol{\beta})$

So $\mathbf{m}$ is $N \times 1$.

Then the average marginal effect of the kth variable is $\frac{1}{N} \sum_{i=1}^{N} m_i$

Question 2

"The interaction effect of a change in age on the income marginal effect at the mean of the data" means

$$\frac{\partial^2 \Pr(y=1|\mathbf{X})}{\partial inc \, \partial age}$$

which yields:

$$\beta_{inc\times age} \cdot \frac{\partial \Phi(\mathbf{X}\boldsymbol{\beta})}{\partial \mathbf{X}\boldsymbol{\beta}} + (\beta_{inc} + \beta_{inc\times age} \cdot X_{age})(\beta_{age} + \beta_{inc\times age} \cdot X_{inc}) \cdot \frac{\partial^2 \Phi(\mathbf{X}\boldsymbol{\beta})}{\partial (\mathbf{X}\boldsymbol{\beta})^2}$$

Where $\dfrac{\partial \Phi(\mathbf{X}\boldsymbol{\beta})}{\partial \mathbf{X}\boldsymbol{\beta}} = \phi(\mathbf{X}\boldsymbol{\beta})$; $\dfrac{\partial^2 \Phi(\mathbf{X}\boldsymbol{\beta})}{\partial (\mathbf{X}\boldsymbol{\beta})^2} = -\mathbf{X}\boldsymbol{\beta} \cdot \phi(\mathbf{X}\boldsymbol{\beta})$