

AAE 637 Lab 6: Math to MATLAB; Statistical Power*

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Key things about translating math to MATLAB:

- MATLAB follows the sequence of operations from top to bottom
- Ultimately, MATLAB is always manipulating numbers, not abstract variables.
- Therefore, a variable must be assigned a numeric value before MATLAB can process it.

Math to MATLAB correspondence			
Operation	Math notation	MATLAB	Returns
Summation	$\sum_{i=1}^n x_i + ay_i$	<code>sum(x + a.*y)</code>	scalar
Vector sum	$\mathbf{x} + \mathbf{y}$	<code>x + y</code>	vector
Indicator function	$(\mathbf{x} + \mathbf{y}) \cdot \mathbb{1}\{\mathbf{z} = 0\}$	<code>(x+y) .* (z==0)</code>	vector
Identity matrix	I_n	<code>eye(n)</code>	matrix
Dot product	$\mathbf{x}' \cdot \mathbf{y}$ OR $\sum_{i=1}^n x_i \cdot y_i$	<code>x'*y</code>	scalar
Max of each column	$[\max\{\mathbf{x}_1\}, \dots, \max\{\mathbf{x}_N\}]$	<code>max(X)</code>	vector
Function gradient	$\left[\frac{\partial f(\boldsymbol{\beta})}{\partial \beta_1} \Big _{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}}, \dots, \frac{\partial f(\boldsymbol{\beta})}{\partial \beta_n} \Big _{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} \right]$	<code>Grad(f, betas, 1)</code>	vector

To check your analytical derivative calculations:

- Using the same point of evaluation, in MATLAB evaluate your analytical derivative function and compute the numerical derivative of the original function. If they are different, something is wrong with your analytical derivative.
- Use wolframalpha.com to double-check your analytical derivatives.

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Large list of MATLAB math functions: <http://www.math.umaine.edu/~hiebler/comp/matlabR.pdf>

Another one: <http://mathesaurus.sourceforge.net/matlab-python-xref.pdf>

Delta method review

To obtain the variance of some function of parameters $g(\hat{\beta})$ when we have the estimate of the variance-covariance matrix of the original parameters $\hat{\mathbf{V}}$, compute:

$$\hat{\mathbf{V}}_{g(\beta)} = \left[\frac{\partial}{\partial \beta} g(\hat{\beta}) \right] \hat{\mathbf{V}} \left[\frac{\partial}{\partial \beta} g(\hat{\beta}) \right]'$$

Once obtained, a Wald hypothesis test can be conducted by:

$$W_n = \left[g(\hat{\beta}) - c \right]' \left(\hat{\mathbf{V}}_{g(\beta)} \right)^{-1} \left[g(\hat{\beta}) - c \right]$$

where the null hypothesis is expressed as $g(\hat{\beta}) = c$.

Drawing the wrong conclusions from hypothesis tests

Failure to reject the null hypothesis does not indicate that the null hypothesis is true.

Example:

Let $\hat{\beta} = 10$, $\widehat{SE}_{\hat{\beta}} = 8$

Say the null hypothesis is $\beta = 0$
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t-stat is then $10/8 = 1.25$, so this null cannot be rejected at the 5% level.

Another null hypothesis is $\beta = 18$

t-stat is then $(10 - 18)/8 = -1$, so this null cannot be rejected at the 5% level.
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If failure to reject the null hypothesis meant that the null hypothesis is true, we could then conclude that both $\beta = 0$ and $\beta = 18$. This is a contradiction.

The underlying issue in the above scenario is an inability to reach conclusions due to insufficient *statistical power*.

Statistical power

The power of a test is:

$$\begin{aligned}
\text{power} &= Pr(\text{reject } H_0 | H_0 \text{ is false}) \\
&= 1 - Pr(\text{do not reject } H_0 | H_0 \text{ is false}) \\
&= 1 - Pr(\text{Type II error})
\end{aligned}$$

		Test Indicates	
		Do not reject H_0	Reject H_0
Truth	H_0 is true	Correct Decision	Type I Error
	H_0 is false	Type II Error	Correct Decision

Type II error \approx “false negative” ; Type I error \approx “false positive”
(Type I error is determined by setting the significance level)

Factors that influence power that are (potentially) under your control:

- Sample size. A higher sample size increases power.
- The chosen significance level (10%, 5%, 1%, etc.).
- Use of a powerful test, e.g. Shapiro–Wilk vs. Kolmogorov–Smirnov test for normal distribution.

Factors that influence power that are not under your control:

- The distance between the true β and the null hypothesis (the true effect size).
- The standard error of the estimator for β .

In practice, most null hypotheses specified by researchers are false. When working with datasets with millions of observations, most regressions reveal that all parameters are statistically different from zero. The magnitude of the difference from zero may be very small, however. Often the real purpose of null hypothesis testing is to reduce the possibility of concluding that the marginal effect is negative when it is actually positive, and vice versa.

Even with high power, we cannot conclude that a parameter takes on any particular value. This is the disadvantage of the Frequentist paradigm, as compared to the Bayesian. The best that can be done is to compute the confidence interval and say that the probability of the confidence interval failing to contain the true parameter value is low.